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Non-perturbative determination of quark masses in quenched lattice QCD with the Kogut-Susskind fermion action

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Abstract

We report results of quark masses in quenched lattice QCD with the Kogut-Susskind fermion action, employing the Reguralization Independent scheme (RI) of Martinelli *et al.* to non-perturbatively evaluate the renormalization factor relating the bare quark mass on the lattice to that in the continuum. Calculations are carried out at $\beta = 6.0$, 6.2, and 6.4, from which we find $m_{ud}^{\overline{\rm MS}}(2{\rm GeV}) = 4.23(29)$MeV for the average up and down quark mass and, with the ϕ meson mass as input, $m_s^{\overline{\rm MS}}(2{\rm GeV}) = 129(12)$MeV for the strange mass in the continuum limit. These values are about 20% larger than those obtained with the one-loop perturbative renormalization factor.

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The values of quark masses are fundamental parameters of the Standard Model which are not directly accessible through experimental measurements. Lattice QCD allows their determination through a calculation of the functional relation between quark masses and hadron masses. For this reason a number of lattice QCD calculations have been carried out to evaluate quark masses, employing the Wilson, clover or Kogut-Susskind (KS) fermion action [1].

An important ingredient in these calculations is the renormalization factor relating the bare lattice quark mass to that in the continuum. While perturbation theory is often used to evaluate this factor, uncertainties due to higher order terms are quite significant in the range of the QCD coupling constant accessible in today's numerical simulations. A non-perturbative determination of the renormalization factor is therefore necessary for a reliable calculation of quark masses, and effort in this direction has recently been pursued for the Wilson and clover fermion actions [2–5].

The need for a non-perturbative determination of the renormalization factor is even more urgent for the KS action since the one-loop correction [6,7] is as large as 50% in present simulations. In this article we report a study to meet this need [8,9] : we calculate the renormalization factor of bi-linear quark operators for the KS action non-perturbatively using the Reguralization Independent scheme (RI) of Ref. [2] developed for the Wilson/clover actions. The results for the scalar operator, combined with our previous calculation of bare quark masses [10], lead to a non-perturbative determination of the quark masses in the continuum limit.

In the RI sheme, the renormalization factor of a bi-linear operator $\mathcal{O}$ is obtained from the amputated Green function,

$$\Gamma_{\mathcal{O}}(p) = S(p)^{-1}\langle 0|\phi(p)\ \mathcal{O}\ \bar{\phi}(p)|0\rangle S(p)^{-1} \tag{1}$$

where the quark two-point function is defined by $S(p) = \langle 0|\phi(p)\bar{\phi}(p)|0\rangle$. The quark field $\phi(p)$ with momentum p is related to the one-component KS field $\chi(x)$ by $\phi_A(p) = \sum_y \exp(-ip\cdot y)\chi(y+aA)$, where $y_\mu = 2an_\mu$, $p_\mu = 2\pi n_\mu/(aL)$ with $-L/4 \leq n_\mu < L/4$ and $A_\mu = 0, 1$.

Bi-linear operators have a form

$$\mathcal{O} = \sum_{yABab} \bar{\phi}^a_A(y)\overline{(\gamma_S \otimes \xi_F)}_{AB}\ U^{ab}_{AB}(y)\phi^b_B(y) \tag{2}$$

where $\overline{(\gamma_S \otimes \xi_F)}$ refers to Dirac (γ_S) and KS flavor (ξ_F) structure [7], and the indices a and b refer to color. The factor $U^{ab}_{AB}(y)$ is the product of gauge link variables along a minimum path from $y+aA$ to $y+aB$. We note that $U_{AB}(y)$ is absent for scalar and pseudo scalar operators as these operators are local.

The renormalization condition imposed on $\Gamma_{\mathcal{O}}(p)$ is given by

$$Z^{\mathrm{RI}}_{\mathcal{O}}(p)\cdot Z_\phi(p) = \mathrm{Tr}[P^\dagger_{\mathcal{O}}\Gamma_{\mathcal{O}}(p)] \tag{3}$$

where $P^\dagger_{\mathcal{O}} = \overline{(\gamma^\dagger_S \otimes \xi^\dagger_F)}$ is the projector onto the tree-level amputated Green function. The wave function renormalization factor $Z_\phi(p)$ can be calculated by the condition $Z_V(p) = 1$ for the conserved vector current corresponding to $\overline{(\gamma_\mu \otimes I)}$. Since the RI scheme explicitly uses the quarks in external states, gauge fixing is necessary. We employ the Landau gauge throughout the present work.

The relation between the bare operator on the lattice and the renormalized operator in the continuum takes the form,

$$\mathcal{O}_{\overline{\mathrm{MS}}}(\mu) = U_{\overline{\mathrm{MS}}}(\mu, p) Z_{\mathrm{RI}}^{\overline{\mathrm{MS}}}(p)/Z_{\mathcal{O}}^{\mathrm{RI}}(p)\mathcal{O} \tag{4}$$

where $U_{\overline{\mathrm{MS}}}(\mu, p)$ is the renormalization-group running factor in the continuum from momentum scale p to μ. We adopt the naive dimensional regularization (NDR) with the modified minimum subtraction scheme ($\overline{\mathrm{MS}}$) in the continuum. The factor $Z_{\mathrm{RI}}^{\overline{\mathrm{MS}}}(p)$ provides matching from the RI scheme to the $\overline{\mathrm{MS}}$ scheme. These two factors are calculated perturbatively in the continuum. For our calculation of the quark mass we apply the relation (4) in the scalar channel in the chiral limit, *i.e*, $1/Z_m^{\mathrm{RI}} = Z_S^{\mathrm{RI}}$.

Our calculations are carried out in quenched QCD. Gauge configurations are generated with the standard plaquette action at $\beta = 6.0$, 6.2, and 6.4 on an 32^4 lattice. For each β we choose three bare quark masses tabulated in Table I where the inverse lattice spacing $1/a$ is taken from our previous work [10].

We calculate Green function for 15 momentum in the range $0.038553 \leq (ap)^2 \leq 1.9277$. Quark propagators are evaluated with a source in momentum eigenstate. We find that the use of such a source results in very small statistical errors of $O(0.1\%)$ in Green functions.

The RI method completely avoids the use of lattice perturbation theory. We do not have to introduce any ambiguous scale, such as q^* [11], to improve on one-loop results. An important practical issue, however, is whether the renormalization factor can be extracted from a momentum range $\Lambda_{\mathrm{QCD}} \ll p \ll O(1/a)$ keeping under control the higher order effects in continuum perturbation theory, non-perturbative hadronization effects, and the discretization error on the lattice. These effects appear as p dependence of the renormalization factor in (4), which should be absent if these effects are negligible.

In Fig. 1 we compare the scalar renormalization factor $Z_S^{\mathrm{RI}}(p)$ with that for pseudo scalar $Z_P^{\mathrm{RI}}(p)$ for three values of bare quark mass am at $\beta = 6.0$. From chiral symmetry of the KS fermion action, we naively expect a relation $Z_S^{\mathrm{RI}}(p) = Z_P^{\mathrm{RI}}(p)$ for all momenta p in the chiral limit. Clearly this does not hold with our result toward small momenta, where $Z_P^{\mathrm{RI}}(p)$ rapidly increases as $m \to 0$, while $Z_S^{\mathrm{RI}}(p)$ does not show such a trend.

To understand this result, we note that chiral symmetry of KS fermion leads to the following identities between the amputated Green function of the scalar $\Gamma_S(p)$, pseudo scalar $\Gamma_P(p)$, and the quark two-point function $S(p)^{-1}$:

$$\Gamma_S(p) = \frac{\partial}{\partial m} S(p)^{-1} \tag{5}$$

$$\Gamma_P(p) = \frac{1}{2m}[\overline{(\gamma_5 \otimes \xi_5)} S(p)^{-1} + S(p)^{-1} \overline{(\gamma_5 \otimes \xi_5)}] \tag{6}$$

We also find numerically that the quark two-point function can be well represented by

$$S(p)^{-1} \sim \sum_{\mu} \overline{(\gamma_\mu \otimes I)} \Sigma_\mu^\dagger(p) i C_\mu(p) + M(p) \tag{7}$$

with two real functions $C_\mu(p)$ and $M(p)$, where $\Sigma_\mu^\dagger(p) = \cos(ap_\mu) - i\overline{(\gamma_\mu\gamma_5 \otimes \xi_\mu\xi_5)} \sin(ap_\mu)$. From (6), (7), and (3) we obtain the relations between the renormalization factors and $M(p)$,

$$Z_S^{\mathrm{RI}}(p) \cdot Z_\phi(p) = \partial M(p)/\partial m \tag{8}$$

$$Z_P^{\mathrm{RI}}(p) \cdot Z_\phi(p) = M(p)/m \tag{9}$$

In Fig. 2 $M(p)$ in the chiral limit obtained by a linear extrapolation in m is plotted. It rapidly dumps for large momenta, but largely increases toward small momenta. Combined with (9) this implies that $Z_P^{\rm RI}(p)$ diverges in the chiral limit for small momenta, which is consistent with the result in Fig. 1.

The function $M(p)$ is related to the chiral condensate as follows :

$$\langle\phi\bar{\phi}\rangle = \sum_p \mathrm{Tr}[S(p)] = \sum_p \frac{M(p)}{\sum_\mu C_\mu(p)^2 + M(p)^2} \tag{10}$$

A non-vanishing value of $M(p)$ for small momenta would lead to a non-zero value of the condensate. Therefore the divergence of $Z_P^{\rm RI}(p)$ near the chiral limit is a manifestation of spontaneous breakdown of chiral symmetry; it is a non-perturbative hadronization effect arising from the presence of massless Nambu-Goldstone boson in the pseudo scalar channel.

While we do not expect the pseudo scalar meson to affect the scalar renormalization factor $Z_S^{\rm RI}(p)$, as indeed observed in the small quark mass dependence seen in Fig. 1, the above result raises a warning that $Z_S^{\rm RI}(p)$ may still be contaminated by hadronization effects for small momenta.

In Fig. 3 we show the momentum dependence of $Z_m(\mu,p,1/a) \equiv U_{\overline{\rm MS}}(\mu,p) Z_{\rm RI}^{\overline{\rm MS}}(p) Z_S^{\rm RI}(p)$ which is the renormalization factor from the bare quark mass on the lattice to the renormalized quark mass at scale μ in the continuum. Here we set $\mu = 2$GeV and use the three-loop formula [12] for $U_{\overline{\rm MS}}$ and $Z_{\rm RI}^{\overline{\rm MS}}$. While $Z_m(\mu,p,1/a)$ should be independent of the quark momentum p, our results show a sizable momentum dependence which is almost linear in $(ap)^2$ for large momenta (filled symbols in Fig. 3).

For small momenta we consider that the momentum dependence arises from non-perturbative hadronization effects on the lattice and the higher order effects in continuum perturbation theory. It is very difficult to remove these effects from our results.

Toward large momenta, however, these effects are expected to disappear. The linear dependence on $(ap)^2$, which still remains, should arise from the discretization error on the lattice, *i.e.*,

$$Z_m(\mu,p,1/a) = m^{\overline{\rm MS}}(\mu)/m + (ap)^2 Z_{\rm H} + O(a^4) \tag{11}$$

with the constant $Z_{\rm H}$ corresponding to the mixing to dimension 5 operators on the lattice.

This relation implies that, if we take a continuum extrapolation of $Z_m(\mu,p,1/a)m$ at a fixed physical momentum p, the discretization error in Z_m is removed. This procedure also removes the a^2 discretization error in the lattice bare quark mass m itself reflecting that in hadron masses.

The momentum p should be chosen in the region where the linear dependence on $(ap)^2$ is confirmed in our results. This region starts from a similar value of $p^2 \approx 3\mathrm{GeV}^2$ for the three β values, and extends to $p^2 \approx 1.9/a^2$, the highest momentum measured. Hence we are able to use only a rather narrow range $3\mathrm{GeV}^2 < p^2 < 6.6\mathrm{GeV}^2$, the upper bound dictated by the value of $1.9/a^2$ for the largest lattice spacing at $\beta = 6.0$.

In Fig. 4 we show the continuum extrapolation for the averaged up and down quark mass at $\mu = 2$GeV. Filled circles are obtained for $p = 1.8$GeV and squares for $p = 2.6$GeV for which the value of Z_m is obtained by a linear fit in $(ap)^2$ employing the filled points in Fig. 3. The bare quark mass [10] is determined by a linear extrapolation of pseudo scalar

meson mass squared in the Nambu-Goldstone channel $\overline{(\gamma_5 \otimes \xi_5)}$ and that of vector meson mass in the VT channel $\overline{(\gamma_i \otimes \xi_i)}$ to the physical point of π and ρ meson masses.

We observe that the continuum extrapolation completely removes the momentum dependence of the quark mass at finite lattice spacings. Furthermore the values are substantially larger than those obtained with one-loop perturbation theory (open circles for $q^* = 1/a$ and squares for $q^* = \pi/a$). Making a linear extrapolation in a^2, our final result in the continuum limit is

$$m_{ud}^{\overline{\rm MS}}(2{\rm GeV}) = 4.23(29){\rm MeV}. \tag{12}$$

where we adopt the value for $p = 2.6$GeV since this is the largest momentum accessible and the momentum dependence is negligible. This value is about 20% larger than the perturbative estimates : 3.46(23)MeV for $q^* = 1/a$ and 3.36(22)MeV for $q^* = \pi/a$. We collect the values of renormalization factor and quark masses in Table II and III.

Applying our renormalization factor to the strange quark mass, we obtain

$$m_s^{\overline{\rm MS}}(2{\rm GeV}) = 106.0(7.1){\rm MeV} \quad \text{for } m_K \tag{13}$$

$$= \ 129(12){\rm MeV} \quad \text{for } m_\phi \tag{14}$$

where we use K or ϕ meson mass to determine the bare strange mass. Results from perturbative estimation are given in Table IV.

The CP-PACS Collaboration recently reported the results [13] $m_{ud}^{\overline{\rm MS}}(2{\rm GeV}) = 4.6(2)$MeV, $m_s^{\overline{\rm MS}}(2{\rm GeV}) = 115(2){\rm MeV}(m_K)$ and $143(6){\rm MeV}(m_\phi)$ from a large-scale precision simulation of hadron masses with the Wilson action. Our values are 10% smaller, which may be due to the use of one-loop perturbative renormalization factor in the CP-PACS analysis.

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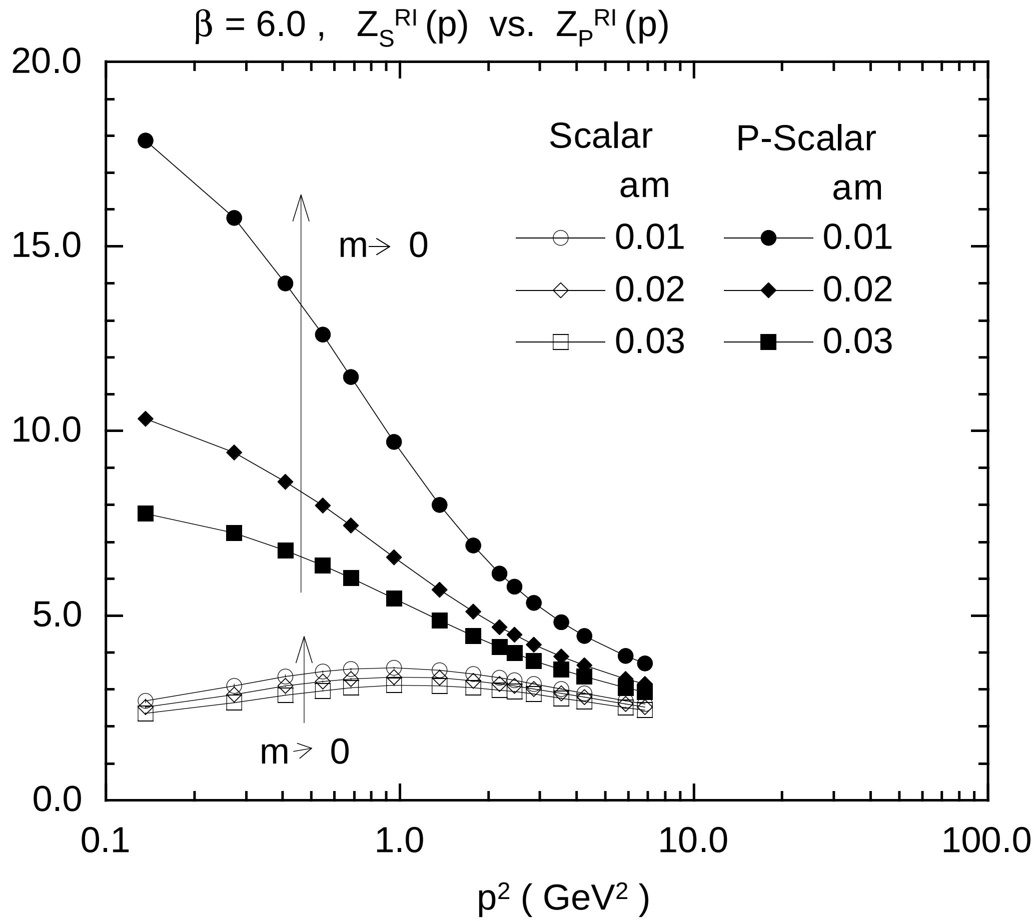

FIG. 1. The renormalization factor for the scalar $Z_S^{\mathrm{RI}}(p)$ and the pseudo scalar $Z_P^{\mathrm{RI}}(p)$.

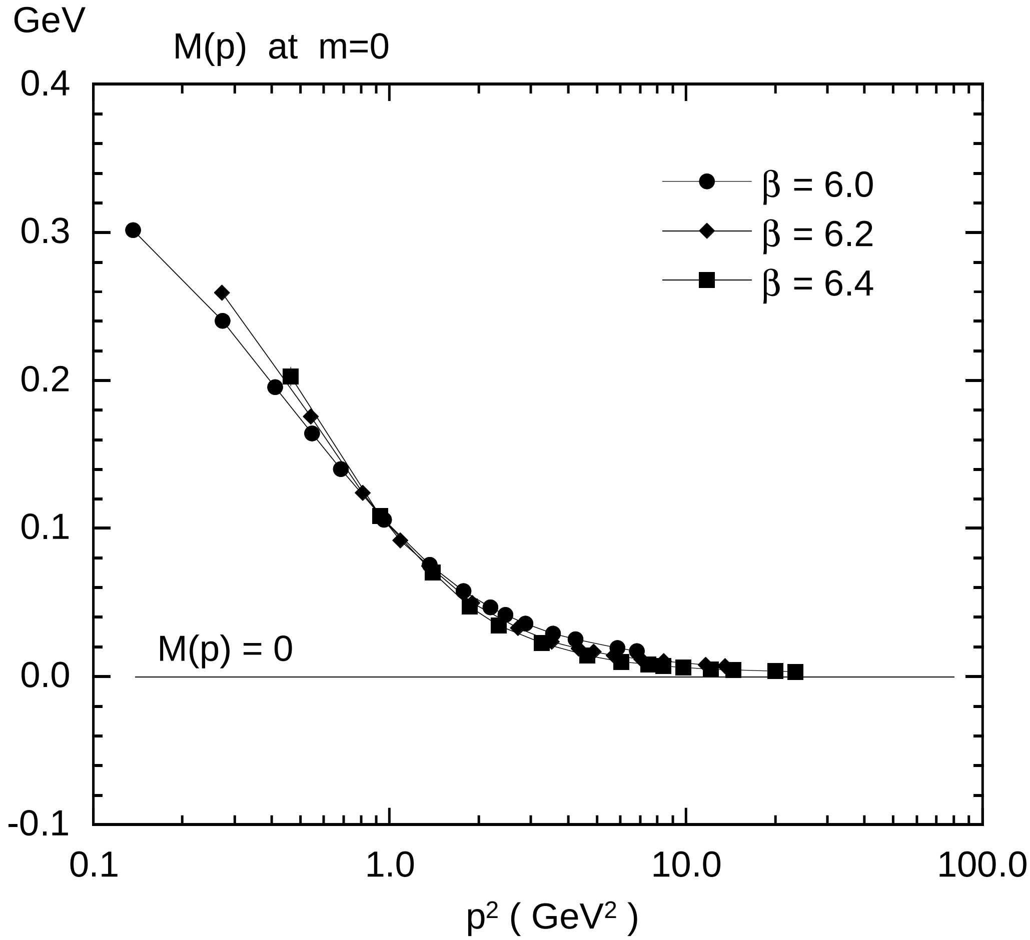

FIG. 2. $M(p)$ in the chiral limit.

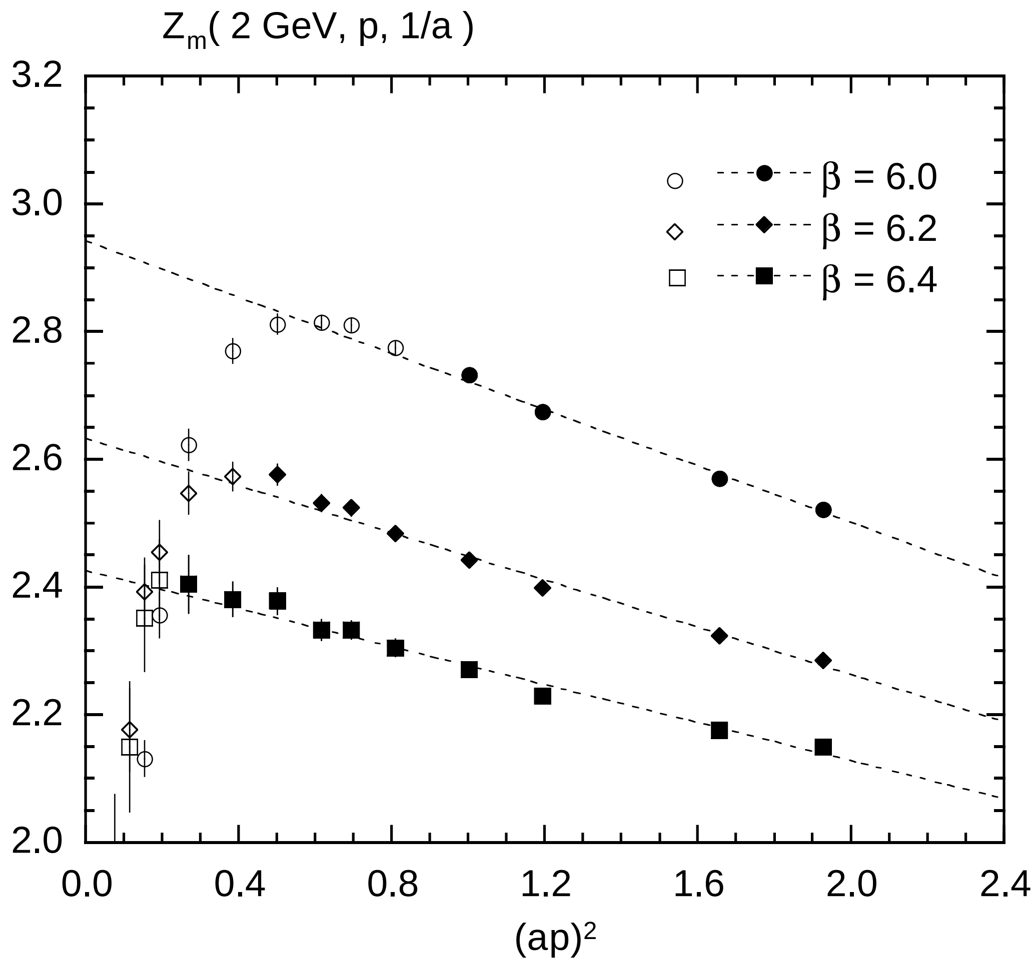

FIG. 3. The ratio $Z_m(\mu, p, 1/a)$ at $\mu = 2$GeV. For each β the filled data points are used for linear interpolation in $(ap)^2$.

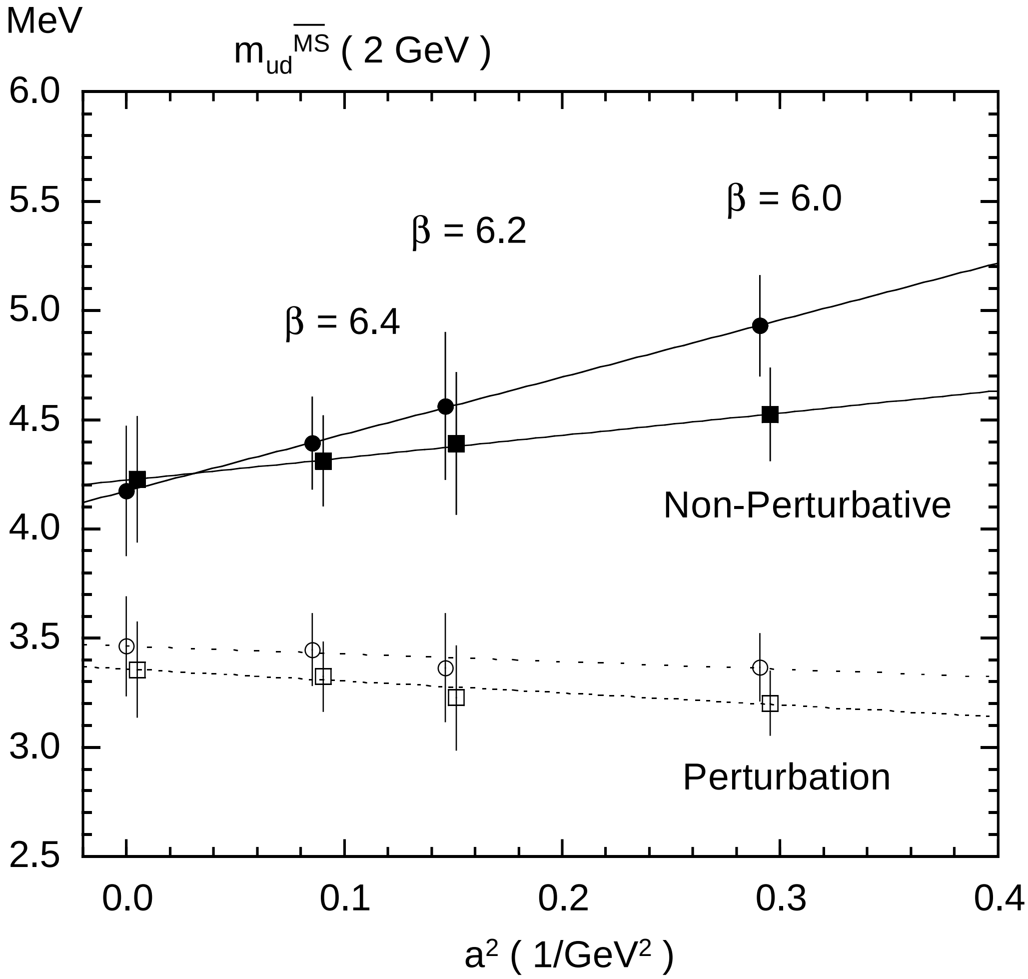

FIG. 4. The final results of the light quark mass $m_{ud}^{\overline{\rm MS}}(2{\rm GeV})$.

TABLES

β	$1/a$(GeV)		am		# conf.
6.0	1.855(38)	0.010	0.020	0.030	30
6.2	2.613(87)	0.008	0.015	0.023	30
6.4	3.425(72)	0.005	0.010	0.020	50

TABLE I. Run parameters.

β	NP.		P.	
	$p = 1.8$GeV	$p = 2.6$GeV	$q^* = 1/a$	$q^* = \pi/a$
6.0	2.735(16)	2.509(18)	1.867	1.776
6.2	2.5451(91)	2.449(10)	1.877	1.800
6.4	2.385(12)	2.339(12)	1.871	1.804

TABLE II. The non-perturbative (NP) and the perturbative (P) renormalization factors of the quark mass $Z_m(\mu, p, 1/a)$ at $\mu = 2$GeV.

β	am_{ud}	NP.(MeV)		P.(MeV)	
	(10^{-4})	$p = 1.8$GeV	$p = 2.6$GeV	$q^* = 1/a$	$q^* = \pi/a$
6.0	9.72(40)	4.93(23)	4.52(21)	3.37(15)	3.20(15)
6.2	6.86(45)	4.56(34)	4.39(32)	3.36(25)	3.23(24)
6.4	5.38(23)	4.39(21)	4.31(21)	3.45(16)	3.32(16)
$a^2 \to 0$		4.17(30)	4.23(29)	3.46(23)	3.36(22)

TABLE III. Final results for $m_{ud}^{\overline{\rm MS}}(2\text{GeV})$ obtained with the non-perturbative (NP) and the perturbative (P) renormalization factors. The lattice bare quark mass am_{ud} is also listed.

β	am_s	NP.(MeV)		P.(MeV)	
	(10^{-2})	$p = 1.8$GeV	$p = 2.6$GeV	$q^* = 1/a$	$q^* = \pi/a$
K input					
6.0	2.44(10)	123.8(5.7)	113.6(5.3)	84.6(3.9)	80.4(3.7)
6.2	1.72(11)	114.4(3.3)	110.1(8.0)	84.5(6.2)	81.0(5.9)
6.4	1.350(57)	110.3(5.2)	108.2(5.1)	86.5(41)	83.4(3.9)
$a^2 \to 0$		104.7(7.4)	106.0(7.1)	86.9(5.6)	84.2(5.4)
ϕ input					
6.0	2.73(15)	138.5(8.2)	127.1(7.5)	94.6(5.4)	90.0(5.2)
6.2	2.04(23)	136(16)	131(15)	100(12)	96(11)
6.4	1.597(97)	130.4(8.4)	128(8.3)	102.3(6.6)	98.7(6.3)
$a^2 \to 0$		128(11)	129(12)	105.5(9.1)	102.2(8.7)

TABLE IV. The final results for the strage quark mass $m_s^{\overline{\rm MS}}(2\text{GeV})$ obtained with K meson or ϕ meson mass to fix the bare strange quark masses am_s.